

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS  
METHODS  
UNIT 1

**SOLUTIONS**

Section Two:  
Calculator-assumed

**Student Name**

**Teacher Name**

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (8 marks)

(a) The variables and are linearly related so that when and when  
. State, with reasoning, whether the relationship between and is an example of direct proportion. (2 marks)

|  |
| --- |
| Solution |
| For direct proportion require that , a straight line through origin.  Since and then and so relationship is an example of direct proportion. |
| Specific behaviours |
| ✓ states relationship is direct proportion (with reasoning)   shows that or similar |

(b) Four points have coordinates and .

(i) If is the midpoint of and , determine the value of the constant and the value of the constant . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of   value of |

(ii) Determine the equation of the line that is perpendicular to and that passes through in the form , where and are integers and .

(4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ slope of   slope of perpendicular line   correct equation of line, any form   correct equation of line in required form |

Question 9 (7 marks)

A circle has equation .

(a) Determine the centre and radius of the circle. (3 marks)

|  |
| --- |
| Solution |
| Centre is and radius is units |
| Specific behaviours |
| ✓ completes the squares   correct centre   correct radius |

Points and lie on the circle above such that the origin is the midpoint of . The line through the centre of the circle to the origin is perpendicular to .

(b) Show that has a length of units. (4 marks)

|  |
| --- |
| Solution |
| Since is the midpoint of , units |
| Specific behaviours |
| ✓ sketch a diagram  ✓ determines (or )   determines   deduces that units using midpoint |

Question 10 (8 marks)

Triangle has sides cm, cm and an area of cm2.

(a) Determine all possible values for the size of angle . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ substitutes into area formula correctly   solves for one correct angle   solves for both correct angles |

(b) Determine, showing use of trigonometry, the largest possible perimeter of triangle .

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
|  substitutes into cosine rule correctly   correct length of   correct perimeter |

(c) Determine, showing use of trigonometry, the size of angle when triangle has the largest possible perimeter. (2 marks)

|  |
| --- |
| Solution |
| (or ) |
| Specific behaviours |
| ✓ substitutes into sine rule/cosine rule correctly   solves for correct angle |

Question 11 (9 marks)

The time of sunrise, hours after midnight, on the day of the year in Esperance is closely modelled by

(a) Use the model to calculate, in hours and minutes, the time of sunrise on February.

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value of   correct time in hours and minutes |

(b) Graph as a function of on the axes below for . (4 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ endpoints   maximum   minimum   smooth curve |

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FXData:

</EFOFEX>

(c) Jill lives in Esperance and is woken by her alarm clock at am every day. How many days will she be awake before sunrise in a year of days? (3 marks)

|  |
| --- |
| Solution |
| when (see dotted line on graph).  when .  Hence awake days through which is days. |
| Specific behaviours |
| ✓ indicates when   indicates correct intervals of days   correct number of days |

Question 12 (3 marks)

The graph below shows information about prisms with the same volume.

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(a) Give **one** example to show that the volume is 24 cm3. (1 mark)

|  |
| --- |
| Solution |
| cm3 |
| Specific behaviours |
| ✓ reads value for and from the graph and determines volume |

The equation of the graph is , where is a constant.

(b) Determine the value of . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
|  correct value |

(c) Circle the correct statement. (1 mark)

|  |
| --- |
| Solution |
| See above |
| Specific behaviours |
|  correct statement |

|  |  |
| --- | --- |
| is directly proportional to | is directly proportional to |
| is inversely proportional to | is directly proportional to |

Question 13 (9 marks)

A function defined by , where and are constants, passes through the points and .

(a) Determine the value of and the value of . (3 marks)

|  |
| --- |
| Solution |
| Solve simultaneously using CAS: |
| Specific behaviours |
| ✓ uses points to form two equations   value of   value of |

(b) Draw the graph of on the axes below, clearly indicating the coordinates of all axes intercepts and equations of any asymptotes. (4 marks)

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|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ location of intercept   equations of both asymptotes   LHS, smooth curve, thru'   RHS, smooth curve, thru' |

(c) State the equations of all asymptotes of the graph of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ equations of horizontal asymptote   equations of vertical asymptote |

Question 14 (9 marks)

<EFOFEX>
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</EFOFEX>The shaded regions shown in the diagram form  
a logo that is to be cut from a plastic sheet.  
  
The boundaries of the shaded regions are  
parts of triangle , where ,  
and arcs of two concentric circles with  
centre and radii of cm and cm.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct conversion |

(a) Express exactly in radians. (1 mark)

(b) Determine the total length of the two curved and three straight cuts that must be made by the cutting machine, giving your answer to the nearest centimetre. (4 marks)

|  |
| --- |
| Solution |
| Length of arcs ():  Length of :  Total lengths of cuts: cm. |
| Specific behaviours |
| ✓ calculates one correct arc length   indicates correct use of cosine rule for   correct length of   correct total length to the nearest centimetre |

(c) Determine the area of the logo, giving your answer to the nearest square centimetre.

(4 marks)

|  |
| --- |
| Solution |
| Area of sector:  Area of segment:  Total area: cm2. |
| Specific behaviours |
| ✓ correct area of sector   indicates correct use of segment area formula   correct area of segment   adds segment and sector to obtain correct total area to the nearest square centimetre |

Question 15 (10 marks)

The diagram below shows the points , and , for .

<EFOFEX>

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(a) Show that the area, , of the triangle is . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct equation for  ✓ expands and shows required result |

(b) Draw a graph of for , on the axes below, clearly labelling all intercepts  
and turning point. (2 marks)

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|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ plots intercepts correctly  ✓ smooth curve through the turning point |

(c) (i) What feature of the graph in part (b) allows you to determine the value of that  
maximises the area? (1 mark)

|  |
| --- |
| Solution |
| Turning point |
| Specific behaviours |
| ✓ states turning point |

(ii) State the value of that maximises the area, and state what is this area. (2 marks)

|  |
| --- |
| Solution |
| Maximum area units2 |
| Specific behaviours |
| ✓ correct value   correct area |

Given , the diagram at the top of page 10 is extended to form a square .

The four vertices of the square lie on the axes as shown below.

<EFOFEX>

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(d) (i) Determine the equation of the line , which is parallel to . (2 marks)

|  |
| --- |
| Solution |
| Hence is |
| Specific behaviours |
| ✓ determines gradient of   correct equation |

(ii) Determine the area of the square . (1 mark)

|  |
| --- |
| Solution |
| units2 |
| Specific behaviours |
| ✓ correct area |

Question 16 (8 marks)

(a) The graph of the quadratic function has roots at and and the range of is . Use an algebraic method to determine . (4 marks)

|  |
| --- |
| Solution |
| or  Axis of symmetry is midway between roots and so turning point at .  Using a root, .  Hence . |
| Specific behaviours |
|  writes equation in turning point form or factorised form with constant  ✓ uses midpoint and range to state turning point   evaluates constant   states |

(b) The area of square is cm2 more than six times the area of square . The difference in the perimeters of the two squares is cm. Determine the least possible area of square , the smaller of the squares. (4 marks)

|  |  |  |
| --- | --- | --- |
| Solution | Alternative Solution |  |
| If side of is cm (the smaller square) and side of is cm:  Solving simultaneously using Classpad:  or  Least area when : | Side of must be cm longer than side of .  If side of is , then side of is .  Hence  Least area when : |  |
| Specific behaviours | Specific behaviours |  |
| ✓ indicates equation using areas   indicates equation using perimeters   solves equations simultaneously   states least area | ✓ indicates difference in the side lengths   forms quadratic equation   solves quadratic equation   states least area |  |

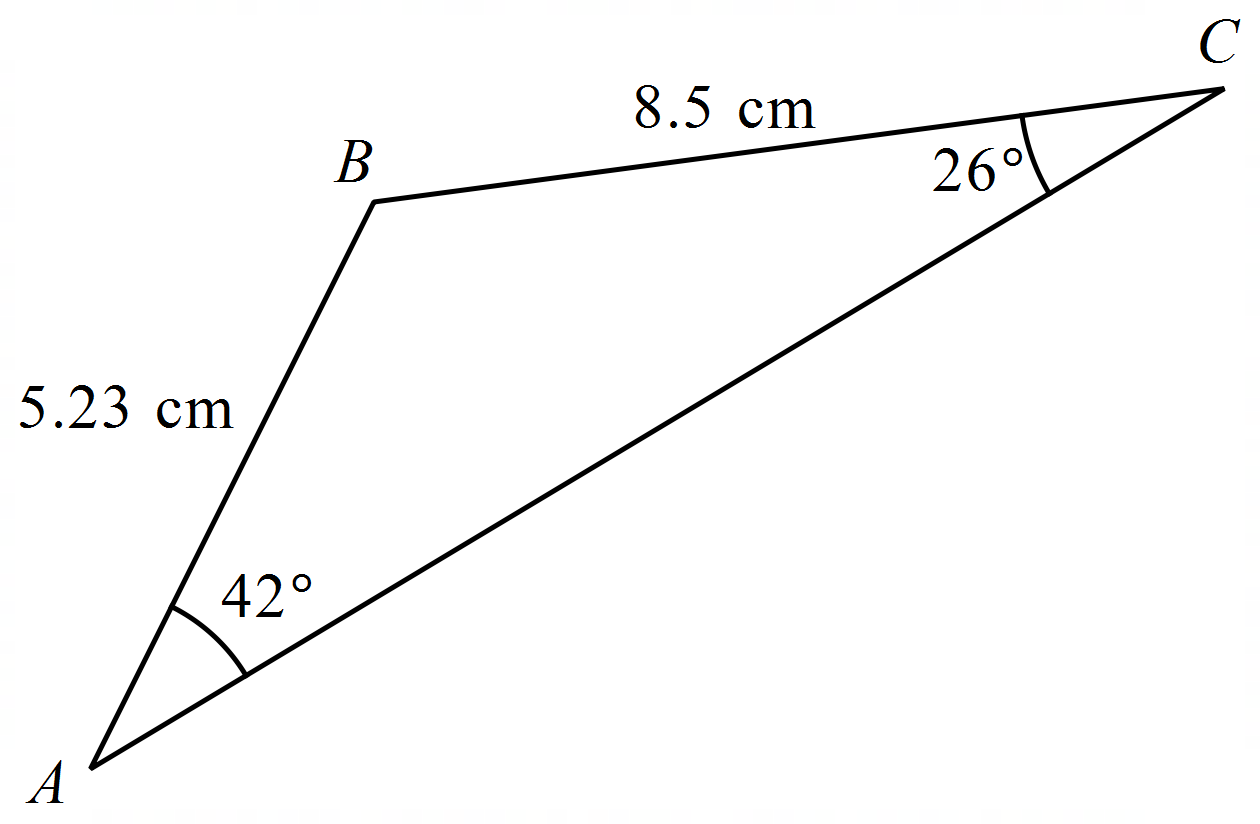
Question 17 (10 marks)

(a) A pilot flies a plane on course to land at Perth Domestic Airport km away. Unfortunately, due to an error, her bearing is out by °. After travelling km she

realises she is off course. How far from the airport is the plane? (3 marks)

|  |
| --- |
| Solution |
| km |
| Specific behaviours |
| ✓ sketch with labelled information   uses cosine rule   correct distance |

(b) Find the area of the triangle shown in the diagram below (not to scale). (2 marks)



|  |
| --- |
| Solution |
| cm2 |
| Specific behaviours |
|  determines included angle   correct area |

(c) Barbara and Leo are geologists who start walking from a mine () in two different directions. Barbara walks for 17 km to a marked beacon on a hill (). Leo walks in a straight line towards a temporary camp (). The angle is 48°. When Barbara reaches the beacon (), she is 13.5 km from the camp (). Given that angle is obtuse, how far does Leo have to walk to reach the camp? (5 marks)

|  |
| --- |
| Solution |
| km |
| Specific behaviours |
| ✓ sketch with labelled information   uses sine rule  ✓ correct   correct equation to determine distance  ✓ correct distance |

Question 18 (7 marks)

A group of students is needed to assist in the school office. The group is to be selected from a class of , in which are boarders and the remainder day students.

(a) Determine the number of different groups that can be selected. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
|  correct number |

(b) Determine the number of different groups that can be selected containing only boarders.

(1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
|  correct number |

(c) Determine the number of different groups that can be selected containing no boarders.

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct use of combinations   correct number |

(d) Determine the number of different groups that can be selected containing at least one boarder and at least one day student. (3 marks)

|  |  |
| --- | --- |
| Solution | Alternative Solution |
| Number of groups with only boarders or only day students:  Hence, | Number of groups with only boarders or only day students: |
| Specific behaviours | Specific behaviours |
| ✓ indicates number with only boarders or only day students  ✓ indicates correct use of combinations   correct number | ✓ indicates number for one case  ✓ indicates number for second case   correct number |

Question 19 (10 marks)

The graph of the cubic polynomial passes through the points , and has a local minimum at .

(a) Use the above information to sketch the graph of on the axes below. (3 marks)

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|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ smooth cubic curve   locates local maximum   locates intercepts |

Let , where and are constants.

(b) Determine the value of each of the constants and . (3 marks)

|  |
| --- |
| Solution |
| Factored form of cubic is  Hence  And so |
| Specific behaviours |
| ✓ correct factored form of cubic   expands cubic   correct value for each constant |

(c) Another cubic polynomial is defined by . Determine the value(s) of the constant so that the graphs of and do not intersect. (4 marks)

|  |
| --- |
| Solution |
| For intersection require .  Hence . For no intersection, this quadratic must have no solution and so discriminant, , must be less than zero.  Hence and so . |
| Specific behaviours |
| ✓ equates cubic equations   obtains quadratic   uses discriminant to form inequality   correct range of values for |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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